NAG Toolbox for MATLAB

f08an

1   Purpose

f08an solves linear least-squares problems of the form

$$\min_x \|b - Ax\|_2 \qquad \text{or} \qquad \min_x \|b - A^\text{H}x\|_2,$$

where A is an m by n complex matrix of full rank, using a QR or LQ factorization of A.

2   Syntax

```
[a, b, info] = f08an(trans, a, b, 'm', m, 'n', n, 'nrhs_p', nrhs_p)
```

3   Description

The following options are provided:

1. If **trans** = 'N' and $m \geq n$: find the least-squares solution of an overdetermined system, i.e., solve the least-squares problem

    $$\min_x \|b - Ax\|_2.$$

2. If **trans** = 'N' and $m < n$: find the minimum norm solution of an underdetermined system $Ax = b$.

3. If **trans** = 'C' and $m \geq n$: find the minimum norm solution of an undetermined system $A^\text{H}x = b$.

4. If **trans** = 'C' and $m < n$: find the least-squares solution of an overdetermined system, i.e., solve the least-squares problem

    $$\min_x \|b - A^\text{H}x\|_2.$$

Several right-hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the m by r right-hand side matrix B and the n by r solution matrix X.

4   References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5   Parameters

5.1   Compulsory Input Parameters

1:   **trans – string**

   If **trans** = 'N', the linear system involves A.

   If **trans** = 'C', the linear system involves A^H.

   Constraint: **trans** = 'N' or 'C'.

2:   **a**(**lda**,∗) **– complex array**

   The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A.

3:     **b(ldb,∗) − complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{m}, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The matrix B of right-hand side vectors, stored in columns; **b** is m by r if **trans** = 'N', or n by r if **trans** = 'C'.

5.2   Optional Input Parameters

1:     **m − int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2:     **n − int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

3:     **nrhs_p − int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrices B and X.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3   Input Parameters Omitted from the MATLAB Interface

lda, ldb, work, lwork

5.4   Output Parameters

1:     **a(lda,∗) − complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $\mathbf{m} \geq \mathbf{n}$, **a** contains details of its QR factorization as returned by f08as.

If $\mathbf{m} < \mathbf{n}$, **a** contains details of its LQ factorization as returned by f08av.

2:     **b(ldb,∗) − complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{m}, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

b contains the solution vectors, x, stored in columns:

if **trans** = 'N' and $m \geq n$, or **trans** = 'C' and $m < n$, elements 1 to $\min(m, n)$ in each column of **b** contain the least squares solution vectors; the residual sum of squares for the solution is

given by the sum of squares of the modulus of elements $(\min(m, n) + 1)$ to $\max(m, n)$ in that column;

otherwise, elements 1 to $\max(m, n)$ in each column of **b** contain the minimum norm solution vectors.

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **trans**, 2: **m**, 3: **n**, 4: **nrhs_p**, 5: **a**, 6: **lda**, 7: **b**, 8: **ldb**, 9: **work**, 10: **lwork**, 11: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, diagonal element i of the triangular factor of A is zero, so that A does not have full rank; the least squares solution could not be computed.

7 Accuracy

See Section 4.5 of Anderson *et al.* 1999 for details of error bounds.

8 Further Comments

The total number of floating-point operations required to factorize A is approximately $\frac{8}{3}n^2(3m - n)$ if $m \geq n$ and $\frac{8}{3}m^2(3n - m)$ otherwise. Following the factorization the solution for a single vector x requires $O\big(\min(m^2, n^2)\big)$ operations.

The real analogue of this function is f08aa.

9 Example

```
trans = 'No transpose';
a = [complex(0.96, -0.8100000000000001), complex(-0.03, +0.96), complex(-
0.91, +2.06), complex(-0.05, +0.41);
    complex(-0.98, +1.98), complex(-1.2, +0.19), complex(-0.66, +0.42),
...
    complex(-0.8100000000000001, +0.5600000000000001);
     complex(0.62, -0.46), complex(1.01, +0.02), complex(0.63, -0.17),
complex(-1.11, +0.6);
     complex(-0.37, +0.38), complex(0.19, -0.54), complex(-0.98, -0.36),
complex(0.22, -0.2);
      complex(0.83, +0.51), complex(0.2, +0.01), complex(-0.17, -0.46),
complex(1.47, +1.59);
             complex(1.08,  -0.28),  complex(0.2,  -0.12),  complex(-
0.0700000000000001, +1.23), complex(0.26, +0.26)];
b = [complex(-2.09, +1.93);
     complex(3.34, -3.53);
     complex(-4.94, -2.04);
     complex(0.17, +4.23);
```

```
      complex(-5.19, +3.63);
      complex(0.98, +2.53)];
[aOut, bOut, info] = f08an(trans, a, b)
```
```
aOut =
  -3.0870                 -0.4885 - 1.1417i    0.3774 - 1.2437i   -0.8552 -
0.7073i
  -0.3270 + 0.4238i    1.5163                  1.3731 - 0.8176i   -0.2509 +
0.8203i
   0.1692 - 0.0798i   -0.4537 - 0.0065i   -2.1713                 -0.2273 -
0.2957i
  -0.1060 + 0.0727i   -0.2734 + 0.0978i   -0.2918 + 0.4888i   -2.3534
   0.1729 + 0.1606i   -0.3236 + 0.1230i    0.2728 + 0.0470i    0.7054 +
0.2515i
   0.2699 - 0.0152i   -0.1646 + 0.3389i    0.5348 + 0.3988i    0.2703 -
0.0727i
bOut =
  -0.5044 - 1.2179i
  -2.4281 + 2.8574i
   1.4872 - 2.1955i
   0.4537 + 2.6904i
  -0.0381 - 0.0273i
  -0.0144 + 0.0483i
info =
         0
```